Multiple-priority impedance control

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***Abstract*— Impedance control is well-suited to robot manipulation applications because it gives the designer a measure of control over how the manipulator to conforms to the environment. However, in the context of end-effector impedance control when the robot manipulator is redundant with respect to end-effector configuration, the question arises regarding how to control the impedance of the redundant joints. This paper considers multi-priority impedance control where a second-priority joint space impedance operates in the null space of a first-priority Cartesian impedance at the end-effector. A control law is proposed that realizes both impedances while observing the priority constraint such that a weighted quadratic error function is optimized. This control law is shown to be a generalization of several motion and impedance control laws found in the literature. The paper makes explicit two forms of the control law. In the first, parametrization by passive inertia values allows the control law to be implemented without requiring end-effector force measurements. In the second, a class of parametrizations is introduced that makes the null space impedance independent of end-effector forces. The theoretical results are illustrated in simulation.**

I. Introduction[1]

Recently, there has been a trend toward building torque-controlled rather than position-controlled manipulators with low viscous and coulomb friction. Manipulators such as the Barrett whole arm manipulator (WAM) [1], the DLR lightweight arm [2], and Robonaut 2 [3] are all torque controlled. For these manipulators, impedance control is an attractive option because of its stability when making or breaking contact with the environment. Nevertheless, relatively little research exists on the subject of multi-priority impedance control analogous to multi-priority position control. In multi-priority position control, low-priority position objectives are realized in the null space of the primary end-effector position objective [4]. Similarly, this paper proposes a controller that regulates manipulator impedance such that a first-priority impedance objective is always met and a second-priority impedance is met to as great an extent as possible without violating constraints associated with the first-priority impedance. This paper focuses on the special case where the first-priority impedance is defined at the end-effector in Cartesian space and the second-priority impedance is defined in joint space. The case where the second-priority impedance is defined in Cartesian space for a separate end-effector is addressed in a companion paper [5].

Multi-priority control in general has been studied extensively. Many early approaches calculated joint velocities that attempt to achieve a second-priority objective (such as avoiding obstacles or manipulator singularities) while achieving a desired end-effector velocity [6], [7], [8]. Chiaverini considers a damped-least-squares version of the control law that is robust to algorithmic singularities [9]. Antonelli provides a Lyapunov analysis demonstrating that the basic approach is stable [10]. Related approaches have been applied in more general contexts [11], [12]. A significant body of work explores the general problem of impedance control in the context of redundant manipulators. Building on Hogan's early work [13], Natale *et. al.* propose a version of the impedance controller that correctly handles angular impedances [14]. Albu-Schaffer and Hirzinger propose a dual-priority impedance architecture where the end-effector impedance and null space joint impedance are controlled separately using a stiffness formulation [15] and Oh *et. al.* propose an impedance control formulation where task and redundant space tasks are dynamically decoupled [16]. Ott *et. al.* demonstrate stability even though the redundant space is non-integrable [17]. Perhaps the most extensive work regarding multi-priority impedance control is by Sentis and Khatib who proposed a multi-priority framework where multiple Cartesian acceleration, force, or impedance objectives can operate with a specified priority [18], [19]. This paper can be viewed as a generalization of that approach to arbitrary second-priority optimization criteria.

The current work was motivated by the need to control the impedance of the Robonaut 2 arms both in the operational space and the redundant space. We propose a control law that minimizes a weighted quadratic optimization criterion defined with respect to the second-priority impedance objective. The resulting control law is stable and that it turns out to be a generalization of several motion and impedance control laws found in the literature [20], [21], [22]. We show that the closed-loop null-space dynamics are independent of end-effector loads whenever the second-priority optimization criterion is identical to the desired second-priority inertia. Finally, we explicitly write the control law for the special case where the desired inertias are all passive and force measurements are not needed to implement the control law. Our approach to the problem is similar to that taken in [23].

II. Background

The dynamic motion of a robot arm with n revolute joints is typically understood in terms of the following generalized

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[1]Portions of this publication have patents pending.

equation of motion:

$$M\ddot{q}+\eta=\tau_a+\tau,$$

where M is the $n\times n$ manipulator inertia matrix, $\ddot{q}$ is an n-vector of manipulator joint accelerations, τ is an n-vector of joint torques resulting from externally applied loads, τ_a is a vector of actuator torques, and η describes the sum of frictional, Coriolis, centrifugal, and gravitational torques [24]. The dependence of these terms on manipulator configuration is implicitly assumed. In order to simplify notation, we introduce the following substitution:

$$\tau_a=u+\eta,$$

such that the equation of motion can be expressed in terms of a command vector, u:

$$M\ddot{q}=u+\tau. \tag{1}$$

It is frequently useful to design controllers defined in operational space coordinates rather than in joint space. While the term "operational space" may refer to any coordinate system relevant to a robot task, it usually refers to the space of positions and orientations of the end-effector represented by a parametrization of $SE(3)$. In this paper, $SE(3)$ is parametrized using exponential coordinates whereby a Cartesian pose is encoded by a 6-vector with the first three numbers describing position and the last three numbers describing orientation using the axis-angle representation [25]. The Cartesian velocity of the end-effector will be represented as a twist and the acceleration as the derivative of twist. Similarly, loads in Cartesian space will be written as wrenches (six-vector that concatenates a force and a moment) [25]. The end-effector Jacobian, J, relates joint velocities, $\dot{q}$, to Cartesian twists at the point of reference (POR), $\dot{x}$: $\dot{x}=J\dot{q}$.

Impedance control can be used to realize a second-order linear impedance in Cartesian coordinates [20], [26]. The goal is to control end-effector inertia and damping as well as stiffness so as to realize the following closed loop behavior:

$$\Omega\ddot{\tilde{x}}+B\dot{\tilde{x}}+K\tilde{x}=f, \tag{2}$$

where Ω, B, and K are the desired Cartesian inertia, damping, and stiffness, respectively. $\tilde{x}$ is the end-effector Cartesian pose error, $\dot{\tilde{x}}=\dot{x}-\dot{x}^*$ is the Cartesian twist error, $\ddot{\tilde{x}}=\ddot{x}-\ddot{x}^*$ is the Cartesian acceleration error, and f is the Cartesian wrench. Re-writing Equation 2 in terms of $\ddot{x}$ and using the fact that $\ddot{x}=J\ddot{q}+\dot{J}\dot{q}$, we have:

$$\ddot{q}=J^{+}\left[\ddot{x}^*+\Omega^{-1}\left(f-B\dot{\tilde{x}}-K\tilde{x}\right)-\dot{J}\dot{q}\right]+N\lambda,$$

where J^{+} denotes the Moore-Penrose pseudo-inverse of the Jacobian, N is the right-null space of J, and λ is arbitrary. Setting λ to zero and substituting into the equation of motion (Equation 1), we have: [26], [23]

$$u=MJ^{+}\left[\ddot{x}^*+\Omega^{-1}\left(f-B\dot{\tilde{x}}-K\tilde{x}\right)-\dot{J}\dot{q}\right]-\tau,$$

which, using the Jacobian torque relationship, $\tau=J^Tf$, becomes:

$$u=MJ^{+}\left[\ddot{x}^*+\Omega^{-1}\left(f-B\dot{\tilde{x}}-K\tilde{x}\right)-\dot{J}\dot{q}\right]-J^Tf. \tag{3}$$

This control law allows the control system to realize an arbitrary second-order end-effector impedance of the form in Equation 2.

III. Multiple priority impedance control

A. Optimization Criterion

First, it must be recognized that a first-priority end-effector Cartesian space impedance and a second-priority joint space impedance are incompatible impedances. Since the joint space impedance completely specifies the behavior of each joint, it is not possible to also realize an arbitrary end-effector impedance simultaneously. Since the two impedance objectives are incompatible, an optimization criterion must be selected that determines the impedance that is actually realized. Following [23], this paper assumes the criterion to be a weighted quadratic function of acceleration. Therefore, in the case of a primary Cartesian space impedance and a subordinate joint space impedance, the controller must optimize a weighted quadratic of the joint accelerations among the accelerations that realize the primary Cartesian space impedance.

Define the second-priority joint space impedance to be:

$$\Omega_j\ddot{q}+\tau^*=\tau, \tag{4}$$

where $\tau^*=B_j\dot{q}+K_j\tilde{q}$. This impedance law is defined in terms of τ, the net torques acting on the joints caused by externally applied loads (wrenches) applied to the manipulator. For example, suppose that there are k loads, $\mathbf{f}_1\ldots\mathbf{f}_k$, in addition to the end-effector wrench, $\mathbf{f}$. Then, then the joint torque generated by these loads is:

$$\tau=J^T\mathbf{f}+\sum_{i=1}^{k}J_i^T\mathbf{f}_i, \tag{5}$$

where J_i is the manipulator Jacobian matrix associated with the i^{th} contact load.

It is possible to optimize for the second-priority joint space impedance of Equation 4 by defining the following weighted quadratic function:

$$\varepsilon=(\ddot{q}-\ddot{q}_{des})^TW(\ddot{q}-\ddot{q}_{des}), \tag{6}$$

where the desired joint accelerations, $\ddot{q}_{des}$, are taken to be those required by Equation 4. As a result, the optimization criterion is: $\varepsilon=z^Tz$ where

$$z=W^{\frac{1}{2}}\left(\ddot{q}-\Omega_j^{-1}(\tau-\tau^*)\right). \tag{7}$$

The multi-priority control objective is to minimize Equation 7 subject to realizing the first-priority Cartesian impedance of Equation 2.

B. Control law

Equation 2 defines the first-priority impedance for the end-effector in Cartesian space:

$$\Omega\ddot{\tilde{x}}+f^*=f,$$

where $f^* = B\dot{\tilde{x}} + K\tilde{x}$. Re-writing this as a constraint on joint accelerations, we have:

$$J\ddot{q} = \ddot{x}^* + \Omega^{-1}(f - f^*) - \dot{J}\dot{q}. \quad (8)$$

We must find joint accelerations that minimize ε from among the solutions to Equation 8. In order to accomplish this, solve Equation 7 for $\ddot{q}$,

$$\ddot{q} = W^{-\frac{1}{2}}z + \Omega_j^{-1}(\tau - \tau^*), \quad (9)$$

and substitute into Equation 8:

$$JW^{-\frac{1}{2}}z + J\Omega_j^{-1}(\tau - \tau^*) = \ddot{x}^* + \Omega^{-1}(f - f^*) - \dot{J}\dot{q}.$$

This is the constraint equation written in terms of the optimization variable, z. The least-squares solution for z is found by taking the pseudo-inverse:

$$z = (JW^{-\frac{1}{2}})^+ \left[\ddot{x}^* + \Omega^{-1}(f - f^*) - \dot{J}\dot{q} - J\Omega_j^{-1}(\tau - \tau^*)\right].$$

Substituting back into Equation 9, the desired joint acceleration is:

$$\ddot{q} = J_W^+ \left[\ddot{x}^* + \Omega^{-1}(f - f^*) - \dot{J}\dot{q}\right] + N_W \Omega_j^{-1}(\tau - \tau^*), \quad (10)$$

where

$$J_W^+ = W^{-1}J^T(JW^{-1}J^T)^{-1} \quad (11)$$

is the weighted pseudo-inverse for the matrix W, and we define

$$N_W = I - J_W^+ J$$

to be the weighted null space projection matrix. Substituting back into the equation of motion (Equation 1), we have:

$$\begin{aligned} u \quad = \quad & MJ_W^+ \left[\ddot{x}^* + \Omega^{-1}(f - f^*) - \dot{J}\dot{q}\right] \\ & + MN_W \Omega_j^{-1}(\tau - \tau^*) - \tau. \quad (12) \end{aligned}$$

It can be verified that Equation 12 realizes the primary end-effector impedance (Equation 2) regardless of the value of the weighting matrix, W. However, the choice for W in the optimization criterion does affect how the subordinate joint space control law is realized. The effect of the second-priority impedance can be analyzed by considering the closed-loop behavior in the weighted null space. Multiplying both sides of Equation 10 by N_W, we have:

$$N_W \ddot{q} + N_W \Omega_j^{-1}\tau^* = N_W \Omega_j^{-1}\tau. \quad (13)$$

Note the similarities between the above and the desired second-priority impedance of Equation 4. Essentially, Equation 12 realizes the second-priority joint space impedance in the range space of N_W.

C. Relationship to the literature

Equation 12 can be viewed as a generalization of several standard motion and impedance control laws found in the literature. First, consider the case where it is assumed that no external loads are applied at all ($\tau = 0$), the desired Cartesian inertia is identity ($\Omega = I$), and the null space term is ignored. Then this control law reduces to inverse dynamics control [27], [21]:

$$u = MJ^+ \left[\ddot{x}^* - f^* - \dot{J}\dot{q}\right],$$

where the stiffness and damping terms in f^* are interpreted as PD gains on position error.

Second, suppose that no external loads are applied, the desired Cartesian inertia is identity ($\Omega = I$), the desired second-priority joint-space inertia is set to the passive value ($\Omega_j = M$), and the weighting matrix is set of the passive inertia matrix ($W = M$). In this case, note that:

$$\begin{aligned} MJ_W^+ \quad &= \quad MM^{-1}J^T(JM^{-1}J^T)^{-1} \\ &= \quad J^T\Lambda, \end{aligned}$$

where

$$\Lambda = (JM^{-1}J^T)^{-1} \quad (14)$$

is the passive manipulator inertia described in end-effector Cartesian coordinates. Also note that:

$$\begin{aligned} MN_M M^{-1} \quad &= \quad M(I - M^{-1}J^T(JM^{-1}J^T)^{-1}J)M^{-1} \\ &= \quad (I - J^T(JM^{-1}J^T)^{-1}JM^{-1}) \\ &= \quad N_M^T. \end{aligned}$$

Then, Equation 12 becomes Khatib's operational space controller (also known as the Gauss controller) [22], [21]:

$$u = J^T\Lambda\left(\ddot{x}^* - f^* - \dot{J}\dot{q}\right) - N_M^T\tau^*.$$

Next, assume that all external loads are applied to the end-effector ($\tau = J^T\mathbf{f}$), the weighting matrix is set to identity ($W = I$), and the null space term is ignored. Then Equation 12 reduces to a variant of the impedance control law of Equation 3 [26]:

$$u = MJ^+ \left[\ddot{x}^* + \Omega^{-1}(f - f^*) - \dot{J}\dot{q}\right] - J^T f,$$

where it has been assumed that all external loads are applied at the end-effector.

Finally, suppose that all external loads are applied to the end-effector ($\tau = J^T\mathbf{f}$), the weighting matrix is set to the passive inertia matrix ($W = M$), and the null space term is ignored. Then, Equation 12 reduces to Hogan's impedance control law [20]:

$$u = J^T\Lambda\left[\ddot{x}^* + \Omega^{-1}(f - f^*) - \dot{J}\dot{q}\right] - J^T f.$$

IV. Analysis

A. Stability

The stability of Equation 12 can be established using a Lyapunov analysis. For the purposes of the analysis, assume that the externally applied loads and the desired accelerations are zero. Define the Lyapunov function to be:

$$V = \dot{x}^T\dot{x} + \tilde{x}^T\Omega^{-1}K\tilde{x} + \dot{q}^T N_w^T N_w \dot{q} + \tilde{q}^T N_w^T N_w \Omega_j^{-1}K_j\tilde{q}. \quad (15)$$

Since V spans the entire joint space, demonstrating that $\dot{V}$ is negative semi-definite is sufficient to show stability. The gradient is:

$$\dot{V} = 2\dot{x}^T\ddot{x} + 2\tilde{x}^T\Omega^{-1}K\dot{x} + 2\dot{q}^T N_w^T N_w \ddot{q} + 2\tilde{q}^T N_w^T N_w \Omega_j^{-1}K_j\dot{q}.$$

We solve for $\ddot{x}$ and $\ddot{q}$ by substituting Equation 12 into the system dynamics (Equation 1). Substituting these into the above equation for $\dot{V}$, we get:

$$\dot{V} = -2\dot{x}^T\Omega^{-1}K\dot{x} - 2\dot{q}^T N_w^T N_w \Omega_j^{-1} K_j \dot{q},$$

which is negative semi-definite. Since $\tilde{q} \neq 0$ with $\dot{q} = 0$ is not an equilibrium configuration, we can conclude that the system is globally asymptotically stable at $\tilde{x} = 0$ and $N_w\tilde{q} = 0$ with $\dot{q} = 0$.

B. Independent null space dynamics

A key issue regarding multi-priority impedance control is how externally applied loads at the end-effector affect the second-priority closed-loop impedance. Equation 13 describes the manipulator dynamics that are orthogonal to the end-effector dynamics. Note that for an arbitrary weighting matrix, W, the dynamics of Equation 13 depend on the forces applied at the end-effector because $N_W\Omega_j^{-1}J^T f$ may be non-zero. This also has a steady-state effect because Equation 13 predicts a joint error of $\tilde{q} = K_j^{-1}J^T f$ when $\ddot{q}$ and $\dot{q}$ are zero.

However, an interesting case occurs in Equation 13 when $W = \Omega_j$. In this case, we have:

$$\begin{aligned}\Omega_j N_{\Omega_j}\Omega_j^{-1} &= \Omega_j(I - \Omega_j^{-1}J^T(J\Omega_j^{-1}J^T)^{-1}J)\Omega_j^{-1} \\ &= (I - J^T(J\Omega_j^{-1}J^T)^{-1}J\Omega_j^{-1}) \\ &= N_{\Omega_j}^T.\end{aligned}$$

Therefore, when $W = \Omega_j$, Equation 13 can be re-written as:

$$N_{\Omega_j}^T\Omega_j\ddot{q} + N_{\Omega_j}^T\tau^* = N_{\Omega_j}^T\tau. \quad (16)$$

While the above is similar to Equation 13, note that τ directly multiplies through the transpose of the weighted null space matrix. As a result, note that loads applied to the end-effector are projected to zero:

$$\begin{aligned}N_{\Omega_j}^T\tau &= N_{\Omega_j}^T J^T f \\ &= 0.\end{aligned}$$

Therefore, when $W = \Omega_j$, manipulator dynamics are independent of end-effector loads in the column space of $N_{\Omega_j}^T$. This form of Equation 12 is useful in force and impedance control because unexpected end-effector loads will be decoupled from motion in the inertia-weighted manipulator null space. For example, suppose that the second-priority impedance law is parametrized with the passive manipulator inertia but the weighting matrix is set to identity. Then, end-effector loads will result in null space motions. However, if the weighting matrix in Equation 12 is set to the passive inertia matrix, then coupling disappears. Khatib makes a similar point with regard to the Gauss controller [22]. However, the Gauss controller exclusively sets the weighting matrix to the passive manipulator inertia. The above indicates that similar advantages can be obtained anytime the weighting matrix is equal to the second-priority inertia matrix.

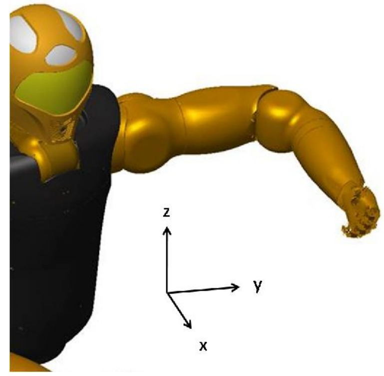


Fig. 1: Simulation of the Robonaut 2 arm. The simulations applied a 20N end-effector (palm of the left hand) force in the positive z direction.

C. Impedance control implementation without end-effector load measurements

In some situations, it may be inconvenient to require the presence of a load cell to implement the control law. This sensing requirement can be circumvented by setting the inertia matrices in the desired impedances to passive values. In particular, recall that Λ is the passive manipulator inertia described in Cartesian coordinates at the end-effector (Equation 14). Then, by setting $\Omega = \Lambda$ and $\Omega_j = M$, the control law of Equation 12 becomes:

$$\begin{aligned}u &= MJ_W^+JM^{-1}J^T(f - f^*) + MJ_W^+\left(\ddot{x}^* - \dot{J}\dot{q}\right) \\ &\quad + MN_WM^{-1}(\tau - \tau^*) - \tau.\end{aligned}$$

If $J^T f$ in the first term above is replaced with τ, the joint torques resulting from externally applied torques, then τ cancels out of the control law entirely and we are left with:

$$\begin{aligned}u &= -MJ_W^+JM^{-1}J^T f^* + MJ_W^+\left(\ddot{x}^* - \dot{J}\dot{q}\right) \\ &\quad - MN_WM^{-1}\tau^*. \end{aligned} \quad (17)$$

Solving for $\ddot{x}$, we have:

$$\Lambda\ddot{\tilde{x}} + f^* = J_M^{+^T}\tau. \quad (18)$$

Although Equation 18 is not identical to the desired first-priority Cartesian impedance (Equation 2), it is a close approximation that does not use independent force or acceleration sensing. When all loads are applied at the end effector, Equation 18 reduces to Equation 2 ($J_M^{+^T}J^T f = f$). When loads are also applied elsewhere on the manipulator, $J_M^{+^T}\tau$ finds a least-squares estimate of the end-effector load.

V. Simulations

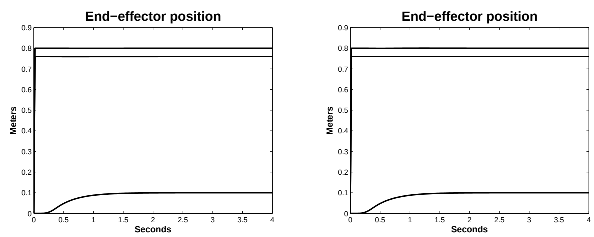
Two instances of Equation 17 with different optimization weighting matrices are simulated. The first sets W to identity:

$$u = -MJ^+JM^{-1}J^T f^* - MJ^+\dot{J}\dot{q} - MNM^{-1}\tau^*. \quad (19)$$

We refer to this controller as the “minimum-acceleration” impedance controller because it minimizes squared acceleration error measured with respect to the second-priority

Link	Length	Mass	I_{xx}	I_{yy}	I_{zz}
upper arm	0.38m	6.8Kg	0.082	0.082	0.0082
forearm	0.35m	5.5Kg	0.056	0.056	0.0056
palm	0.2m	2Kg	0.0004	0.0004	0.00004

TABLE I: Approximate dynamic parameters of Robonaut 2 arm used in the simulations. I_{zz} is the moment of inertia about the link axis. I_{xx} and I_{yy} are the remaining two orthogonal moments of inertia.



(a) (b)

Fig. 2: Simulation of the minimum-acceleration controller, (a), and the Gauss principle controller, (b), responding to a step force applied to the end-effector at $time = 0.1$ seconds. Each plot shows the x, y, and z coordinates of the end-effector as a function of time for the corresponding controller.

impedance objective. The second sets W to the passive inertia matrix ($W = M$):

$$u = -J^T f^* - J^T \Lambda \dot{J}\dot{q} - N_M^T \tau^*. \tag{20}$$

We refer to this controller as the "Gauss principle" impedance controller because when the optimization criterion is parametrized by M, it corresponds to Gauss' Principle of Least Constraint [28].

These evaluations used a dynamic simulation of the Robonaut 2 arm (dynamic parameters were approximated). The arm was approximated by three links, illustrated in Figure 1. The most proximal link was actuated by a roll-pitch-roll shoulder. The elbow was a pitch joint followed by a roll joint. The third link simulated a palm actuated by a co-located pitch-yaw combination. The equations of motion were generated using Autolev [29] based on the dynamic parameters in Table I where, for the purposes of calculating moment of inertia, the links were treated as long rods. Numerical integration of simulation parameters was performed using the Matlab *ode*45 variable step solver.

A. Simulation 1

The first simulation evaluated the dynamic response of the two control laws in Equations 20 and 19 for a step force input of $20N$ applied to the end-effector in the positive z direction (see Figure 1) at $time = 0.1$ seconds. At $time = 0$ seconds, the simulation started with the manipulator elbow at 90 degrees and the first and third joints aligned. Both control laws were parametrized in accordance with a first-priority Cartesian end-effector impedance of

$$\Lambda\ddot{x} + 80\dot{x} + 200\tilde{x} = f,$$

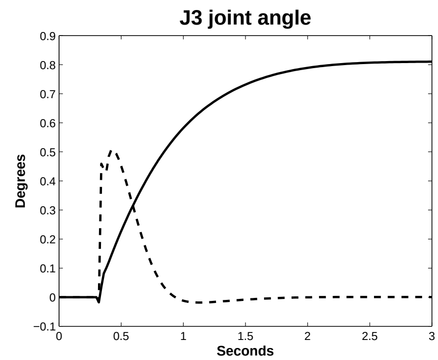


Fig. 3: Simulation of the minimum-acceleration controller, (solid line), and the Gauss principle controller, (dotted line), responding to an external force applied to the end-effector while J3 has a non-zero stiffness. The plot shows J3 motion. Note that the acceleration-based controller does not return J3 to its reference configuration.

and a second-priority joint space impedance of

$$M\ddot{q} + \dot{q} = \tau,$$

where q is measured in radians. Since the second-priority impedance did not have a stiffness term, its main purpose was to stabilize the manipulator by providing null space damping. $\tilde{x}$ was measured with respect to the initial arm configuration. The results, illustrated in Figure 2, indicate that both control laws realize the desired end-effector impedance. This is expected because both control laws have the same dynamics when solved for end-effector acceleration. As a result, we conclude that the two control laws can have significant differences only in their closed-loop null space behavior.

B. Simulation 2

The second simulation was identical to the first except that the following second-priority impedance with a non-zero stiffness was used, $M\ddot{q} + \dot{q} + 25\Sigma_3\dot{q} + 200\Sigma_3\tilde{q} = \tau$. As in the first simulation, the arm started at a joint configuration where the elbow was bent at 90 degrees and the first and third joints were aligned. At $time = 0.3$ seconds, a $20N$ force was applied to the end-effector. The results are shown in Figure 3. The end-effector dynamics were similar to what was observed in Simulation 1. The end-effector exhibited an essentially decoupled Cartesian space impedance. Both control laws result in a J3 displacement from equilibrium. Using the Gauss principle law (the dotted line in Figure 3), J3 is temporarily displaced from the reference. With the acceleration-based law (the solid line), J3 also departs from equilibrium. However, note that under the acceleration-based law, J3 does not return to its reference configuration. This difference was predicted by Equations 13 and 16. Writing Equation 13 for the case of the acceleration law (Equation 19), we have:

$$N\ddot{q} + NM^{-1}\tau^* = NM^{-1}J^T f.$$

Since $NM^{-1}J^T f$ may be non-zero, the acceleration-based law is susceptible to the steady state displacement evidenced in Figure 3. The closed-loop null space behavior of the Gauss-based law (Equation 20),

$$N_M^T M\ddot{q} + N_M^T \tau^* = N_M^T J^T f,$$

is not exposed to this steady-state displacement because $N_M^T J^T f$ is always zero.

VI. Conclusion

Multi-priority impedance control is defined as the problem of realizing a first-priority impedance and a second-priority impedance simultaneously where the first-priority impedance is given the priority where conflicts arise. We restrict our attention to the case where the first-priority impedance objective is defined at the end-effector in Cartesian space and the second-priority impedance is defined in joint space. We solve for the control law that realizes the multi-priority impedance where the optimization function is any weighted quadratic function of acceleration. Regardless of the weighting matrix chosen, the multi-priority control law realizes the desired Cartesian impedance. However, we find that when the weighting matrix matches the second-priority desired inertia, then the resulting closed-loop null space impedance is dynamically independent of end-effector loads. We show that the resulting multi-priority control law can be viewed as a generalization of several motion and impedance control laws from the literature.

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